

NUMERICAL INVESTIGATION OF THE POST MORTEM INTERVAL BASED ON THE HEAT TRANSFER BALANCE OF HUMAN BODY

Nouma Youssef^{1,2,*}, Thaljawi Wathek³, Thaljawi Farouk^{4,5,6}

¹Faculty of Medicine of Sfax, ²Forensic Pathology Department, Teaching Hospital of Medenine, Tunisia, ³Applied Mechanics and Systems Research Laboratory, Tunisia Polytechnic School, University of Carthage, La Marsa, Tunisia, ⁴Faculty of Medicine of Sousse, ⁵Forensic Pathology Department, Regional Hospital of Sidi Bouzid, Tunisia, ⁶Mechanical Department at Higher Institute of Technological Studies of Sidi Bouzid, Tunisia

Abstract: One of the most important results that can be determined by a forensic doctor is the post-mortem interval (PMI) of a corps. Numerous experimental methods are often used but the error rate remains high. In this work, we try a new theoretical method based on heat transfer to minimize the error of traditional methods. We share in this paper the preliminary results. First of all, a simple model of the corps is used and a heat transfer conservation equation is established. In the second part, numerical results by the application of Euler explicit method are discussed.

Keywords: heat transfer, post mortem interval, numerical method.

INTRODUCTION

The human body is a thermal machine capable of producing heat through the metabolic reactions. The effort applied by the human body, tends to increase the metabolism, this energy created is not only dissipated in the form of work, but a significant part will be supplied to the external environment through the skin and clothing by convection, conduction and radiation. This allows us to say that the human corps is considered here as a homogeneous system with a source of internal heat production. The internal temperature of the human body is around 37°C and its maintenance around this value imposes the existence of a thermal balance between the human body and its environment. That is to say, the energy produced and heat transfers to the outside must compensate its storage. The heat balance equation involves three types of items: Heat Generation, Heat Transfer and Heat Storage. The body's metabolism, which generates energy, allows the body to carry out different muscular activities and the rest of the energy will be transferred outside in the form of heat. In the case of death, the thermoregulatory system is no longer functional due to problems that affect the hypothalamus, so what happens is

that the body temperature is no longer constant, it will be increased or decreased until it will be equal to the external temperature. Around this subject, several researchers have tried to find a good thermal model that describes the relationship between human beings and the external environment. Imrich Barta and László Bánhidi (1) established the comfort equation describing the thermal relation between human body and human requirements such as work and buildings. Isawa K. (2) analyzed changes in multiple components of the human body thermal exchanges and the influence of different parameters on the energy balance was mentioned. The aim of this study is to contribute to this field by numerically investigating the Delay Post Mortem (DPM)—the time required for the internal temperature to reach a reference value.

Modeling of the human corps

In the forensic medicine, the prediction of the PMI depends on the initial value of the temperature of the corps at the moment of death (37°C) and the value of the temperature at the moment of discovering of the corps.

The main idea is to look for a relation between these two parameters and the PMI. This relation is obtained by putting the heating transfer plan between the corps and

*Correspondence to: Youssef NOUMA (MD), Forensic pathology department, Teaching Hospital of Medenine, Faculty of Medicine of SFAX, Bd Majida Boulila, Sfax 3029, Tunisia, E-mail: docyoussef@live.fr

the external environment. Therefore, a simple model of the corps is needed to establish the mathematical relation. Generally, the value of the temperature at the moment of discovering of the corps is obtained by putting a probe into the rectum, so only a part of the body (rectum) is needed to be modeled by a simple geometric form (cubic form).

Heat transfer balance

The internal temperature of the human body is around 37°C and its maintenance around this value imposes the existence of a thermal balance between the human body and its environment. That is to say, the energy produced and its storage must be compensated by heat transfers to the outside. In case of death, the thermoregulatory system is no longer functional due to problems affecting the hypothalamus, so what happens is that the body temperature is no longer constant and in most cases, rigor mortis appears after a sudden drop in temperature. Suppose that the metabolic energy is null at the death, and the corps is nude, the model of the heat transfer balance is giving by Fig. 1.

The emitted flux by sun is $\phi_{ES} = \epsilon_S \sigma S_1 (T_s^4 - T_a^4)$.

the absorbed flux is $\phi_{AS} = \alpha_R \phi_{ES}$.

the reflected flux is $\phi_{RS} = \rho_S \phi_{ES}$.

the emitted flux by rectum is $\phi_{ER} = \epsilon_R \sigma (\sum_{i=1}^6 S_i) (T^4 - T_a^4)$.

the convective flux by rectum is $\phi_{CR} = h (\sum_{i=1}^6 S_i) (T - T_a)$.

the conductive flux by rectum is $\phi_{COR} = \frac{\mu}{(\sum_{i=1}^6 D_i)} (T - T_a)$

the storage flux is $\phi_A = \rho c V \frac{dT}{dt}$

The heat balance is written then: $\phi_A + \phi_{AS} = \phi_{ER} + \phi_{CR} + \phi_{COR}$

where ρ , c and V are respectively density, specific heat and perfusion rate of arterial blood, S_i are the appropriate heat transfer surfaces, D_i is the appropriate heat transfer thickness, α_R is the rectum absorptivity, ϵ_S is the sun emittance, σ is the Boltzmann constant T_s , T_a and T are respectively the sun, the rectum and the external temperature. In case of death, the metabolism is neglected to simplify the model.

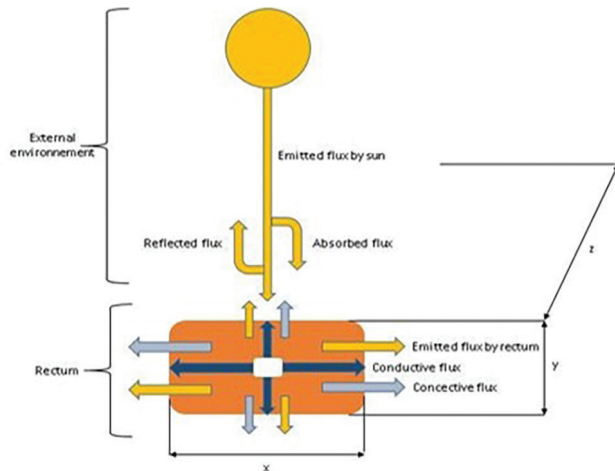


Figure 1. Simple model of the heat balance of the rectum.

Numerical method

As the governing equation of the present heat transfer balance is a differential first order equation, a numerous numerical methods can be used to solve it, in this work, Euler explicit method is used. This method suppose a multiple $n+1$ time points between t_0 and $t_n = \text{DPM}$, so that the solution at is giving by $T_{(n+1)} = T_{n+pf}(T_n, \text{DPM})$ where p is the time step size and $f(T_n, \text{DPM}) = (-\phi_{AS}^n + \phi_{ER}^n + \phi_{CR}^n + \phi_{COR}^n)$.

Numerical steps are resumed in the following diagram (Fig. 2).

RESULTS

In this section, numerical results are established, system parameters are giving by Silva M (3) as bellow:

$\rho = 1100 \text{ Kg m}^{-3}$, $c = 3300 \text{ JKg}^{-1} \text{ K}^{-1}$, $V = 240 \text{ s}^{-1}$, $h = 40 \text{ Wm}^{-2} \text{ K}^{-1}$, $\mu = 0.47 \text{ Wm}^{-1} \text{ K}^{-1}$, $\alpha_r = \epsilon_s = 1$, $D_1 = 0.001 \text{ m}$, $T_s = 5778 \text{ K}$, $T_a = 301 \text{ K}$, $S_1 = S_2 = S_3 = S_4 = S_5 = S_6 = 0.3 \text{ m}^2$. The effect of initial temperature of the human body on the DPM is presented in Figure 3, five human body had respectively five initial temperature, $T_1 = 295 \text{ K}$ (red line), $T_2 = 294 \text{ K}$ (blue line), $T_3 = 293 \text{ K}$ (green line), $T_4 = 292 \text{ K}$ (brown line) and $T_5 = 291 \text{ K}$ (blue line). According to numerical results shown in Figure 3, the human body which has the smallest initial temperature ($T_1 = 295 \text{ K}$) tends to arrive at the ambient temperature the last, arriving at the ambient temperature, there is no more heat exchange between the body and the external environment. These results may be used by forensic doctors to determine the PMI between initial and the discovering temperature. Another numerical study on the effect of the skin thickness is presented in Figure 4.

Two human body with same initial condition, $T_0 = 295 \text{ K}$, and with two different skin thickness, $D_1 = 0.0001 \text{ m}$ (blue line) and $D_2 = 0.001 \text{ m}$ (red line), at the moment of discovering, the one how had the skin thickness value D_2 had the shortest PMI.

The internal temperature therefore reaches the

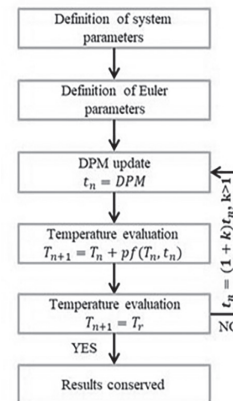


Figure 2. Euler Explicit Algorithm.

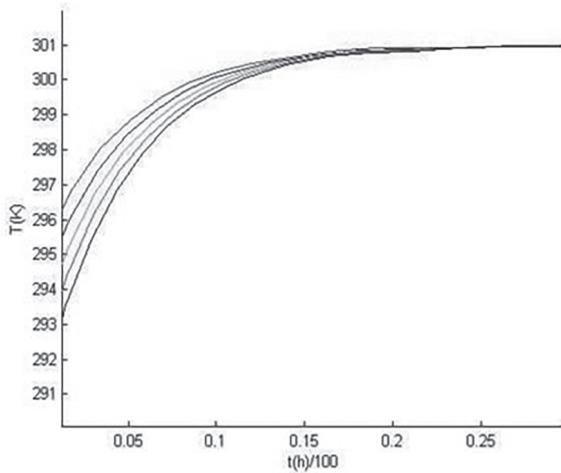


Figure 3. Initial temperature effect on the PMI at the moment of discovering of the human body.

external temperature faster with a very low skin thickness, so, two bodies with the same discovering and internal temperatures with two different skin thicknesses don't have the same PMI. Many other results may be mentioned to deal with the effect of the heat transfer balance and the rectum parameters on the PMI of the human body.

The case studied in this article is very simple and to get closer to reality. We must take into account more complex conditions related to the rectum such as sweat, clothing which will surely modify the heat gain, even the metabolism that we already assumed null can be modeled by a weak source of internal energy. As well as other parameters related to the climate, indeed, the coefficients of convection, conduction and radiation depends on the wind speed, also, the ambient temperature is not constant throughout the day and it can be considered as a sinusoidal function. In this case the heat balance is no longer spontaneous but rather forced. Thus, another formulation of the heat balance where we take in consideration all these changes is obviously recommended. An experimental study must be carried out in order to validate the numerical results. This study will be carried out shortly on real cases taking all ethical considerations. Errors may appear between theoretical analyses and experimental investigations due to several factors in particular the state of the corpses (sweats, drowning, and type of clothes...).

In conclusion, this study has confirmed the critical influence of initial internal temperature and skin thickness on the estimation of PMI through numerical resolution of the heat balance equation. By introducing the concept of DPM, defined as the time required for the internal temperature to reach 300 K under fixed ambient conditions, a clear correlation was established between thermal convergence and forensic timing. Our results demonstrate that lower initial temperatures prolong the

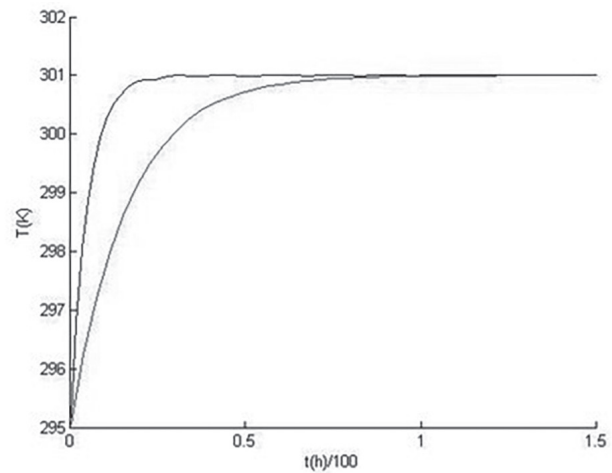


Figure 4. The skin thickness effect on the DPM at the moment of discovering of the human body.

time to thermal equilibration, delaying the PMI estimate. Similarly, increased skin thickness significantly delays this process. Although the present model adopts simplified assumptions, it provides a foundational framework for interpreting thermal behavior in post mortem contexts. To approach realistic conditions, future formulations must incorporate variable ambient temperatures, convective coefficients dependent on wind speed, and other factors such as perspiration, residual metabolic activity and clothing. These additions will transform the heat balance from a spontaneous to a forced system, requiring more sophisticated modeling techniques. Ultimately, experimental validation remains essential to reconcile theoretical predictions with empirical observations, accounting for the variability inherent in real forensic cases. This work lays the groundwork for more comprehensive thermal models capable of supporting accurate and ethically sound PMI assessments.

Conflict of interest

The authors declare that they have no conflict of interest.

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